Algebra: The Key to Student Success, Or Just Another Hurdle?

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Abstract: *The strong emphasis on algebra has led to nationwide school reforms, and it is an integral part of major high school curricula changes that have taken place in recent years. In this article we try to determine if there is a basis for the underlying belief that algebra equips students with the skills necessary to succeed in college. Based on the existing literature, it follows that college algebra may be an indicator of student success in college, but there is no evidence that this subject contributes to that success. There is no evidence of the context or structure of algebra aiding students in developing or even improving any of their cognitive skills, but only their content knowledge. While it is not impossible that this could be the case in a particular situation if the mathematics curricula were constructed with this in mind, no such studies were found in the literature.*

Keywords. *algebra, educational policy, curriculum*

1 Introduction

The story of algebra, its significance, and most importantly, students' attitudes toward this subject may be cleverly summed up in a quote from an episode from an American sports drama, *Friday Night Lights* (2012):

Tyra Collette: Just wish I could build a time machine, and go back, and shoot whoever it is that invented algebra, that's for sure.

Landry Clarke: Well, see, that's kind of a Catch-22, though, because in order to invent that time machine you may need to use algebra.

While achievement in mathematics has been recognized as an essential requirement for progress and success in STEM fields, knowledge of algebra has been proposed as a necessary condition for overall success in mathematics. The highly accentuated emphasis on algebra in secondary and higher education has led to nationwide school reforms, and it has been an integral part of major high school curricula changes that have taken place in recent years. The importance of this subject has been inflated to the point of being equated to overall success in college. Simply put, success in algebra has been seen as an indicator of success in college, and it has been suggested to have strong ties to graduation rates (Parker, 2005).

The story algebra instruction does not, however, begin in college, but rather many years earlier, in the early grades. We explore the story of this subject while probing into the belief that everyone should take algebra. This notion carries itself through high school and into higher education. Its underlying assumption pertains to a relationship between success in algebra and students' overall academic achievement.

In this article we try to identify if there is a basis for the underlying belief that algebra equips students with the skills necessary to succeed in college. By looking at the studies focusing on algebra, both in K-12 and higher education, we seek to identify algebra as either a predictor of college success or a teachable and necessary skill. In the latter case, the requirement of algebra, even in college majors that are not STEM-based, would be justified as it would result in students acquiring skills transferable to other disciplines. However, if the subject itself does not provide students with skills (other than algebraic ones) that positively contribute to a student's overall academic achievement, then algebra is merely a predictor of such success.

Further, in this work we examine if the belief that success in algebra equals student academic success in secondary and higher education is based on satisfactory quantitative and qualitative research. Another possibility that is considered is that this belief is entirely or in part based on studies that do not correct for the fact that students who do take and succeed in more challenging courses, including algebra, are often the most motivated and high-achieving students. We explore this issue within the existing literature, identifying a possible rationale for the widely accepted stance of "algebra for everyone." Furthermore, we study the popular belief that there is a causal relationship between the success in algebra and college graduation rates, and how well supported this belief is by the existing literature.

2 Algebra for Everyone?

The need for algebra is not as obvious in daily life as is the need for arithmetic or even statistics. In reality, even experts believe that people can function adequately in every aspect of life without algebra (Usiskin, 1995). However, while certain functions can be performed without the aid of algebra, knowledge of the subject certainly allows individuals to work more efficiency. Algebra is the language of generalization and relationships between quantities that allows a person to answer all the questions of a certain type at one time (Usiskin, 1995), and this is the skill that is seen as being valuable regardless of the area of the study in which it is applied.

Mathematics, or more specifically, algebra is a gateway to the study of any STEM discipline that students may wish to embark on while in college. It provides or denies a ready access to technically oriented vocational schools, or simply any technically skilled job. This is mainly the reason for encouraging, and recently requiring (in many states), all students to enroll in algebra in the ninth grade. By doing so, students have an opportunity to complete a high school mathematics sequence ending in calculus (Stein, Kaufman, Sherman, & Hillen, 2011). Although the intentions are noble, just how effective are these changes in curriculum, and what is their impact on an overall student academic success? For example, it has been shown that students who take algebra in high school gain in mathematics achievement, regardless of their prior math knowledge; however low-ability students do not achieve as much from algebra as their higher-ability counterparts (Gamoran & Hannigan, 2000).

In 1997, the Chicago Public Schools (CPS) implemented a policy requiring all students to complete a college preparatory curriculum. This included a requiring all students to start taking algebra in ninth grade, instead of having an option of enrolling in remedial math courses for lower-ability students (Allensworth, Nomi, Montgomery, & Lee, 2009). It appears that this policy was at least in part motivated by the same issues that led to the ACT (2008) reports demonstrating that students who complete more rigorous, college preparatory, high school curricula end up performing significantly better on standardized tests. What these reports do not seem to acknowledge, however, is self-selection bias - namely, that higher-achieving students may be more motivated to embark on college preparatory curricula in high school and also prepare more effectively for standardized tests than lower-achieving students. A longitudinal and thorough investigation of the CPS policy revealed results that might not necessarily align with the belief that algebra in high school was beneficial for everyone. For example, students who were required to take algebra starting in ninth grade (according to the implemented policy) were not more likely to attend college, and they were not more likely to pursue math courses beyond the required algebra ones (Allensworth et al., 2009). Surprisingly, higher achieving students were now less likely to go to college after completing the new, college preparatory curriculum (Mazzeo, 2010). Unfortunately, the researchers in this study did not provide any plausible explanations of these outcomes, especially the ones pertaining to higher achieving students.

A lack of students pursuing courses beyond algebra is especially significant considering that the highest mathematics course completed in high school remains a strong predictor of bachelor degree completion (Zelkowski, 2010). If the importance of algebra is in student preparation for college, and if the required course did not seem to result in an increase in the number of students pursuing higher education, one must then question the claim that algebra is essential for all students, particularly for those who intend to pursue non-mathematics careers. If the motivation behind "algebra for all" policies is to "push" students toward STEM disciplines, then requiring completion of algebra, a class that some students may not be academically suited for, may not be the best approach. "If students are earning D's in their courses, can we really expect the content that they are barely learning to matter?" (Allensworth & Nomi, 2009, p. A-5).

Studies that focus on the benefit of enrolling in higher level courses often do not correct for self-selection bias, which may make analysis of their results somewhat questionable. Interpretation of high mathematics achievement rates and even graduation rates of students who voluntarily completed higher level mathematics courses needs to be carefully made because "the students who choose to take high level classes are often the most motivated and high-achieving students in their schools" (Mazzeo, 2010, p.2). Chicago Public Schools' mandatory college preparatory curriculum provided findings independent of students' motivation to enroll in higher level courses, because once the policy was implemented, every student was required to enroll in an algebra course (Allensworth & Nomi, 2009). However, once the selection bias was controlled, the expected outcomes of this policy were not achieved. For example, the policy was expected to generate improvement in student learning and college readiness, which did not occur. Quite the opposite, it appeared that mandatory curriculum had some negative outcomes. Mathematics grades for lower-skill students declined, and these students were more likely to fail first-year algebra (Allensworth & Nomi, 2009). While lower-skill students were now more likely to earn algebra credit, their overall grade point average declined (Mazzeo, 2010).

Average-ability students seemed to be the most affected by this policy. Their math grades plummeted by 8.9% and their average absenteeism rate increased by 1.6 days (Allensworth & Nomi, 2009). The researchers examining the outcomes of Chicago Public School curriculum reform acknowledged that this policy was focused on curricular content, and it did not address how curricular changes might affect instruction and teaching environment. Additionally, while this policy guaranteed content exposure to all students, it did not seek to improve student engagement which was an integral part in overall student academic achievement (Allensworth & Nomi, 2009). The outcomes of this new reform may be interpreted to imply that requiring all students to take algebra would result in an exposure to the content and opportunity to earn a credit for the course. However, those students who are not adequately prepared for this course may be more likely to fail the class, have to repeat it, and hence experience a delay in their timely progress toward graduation. Therefore, a widely accepted belief that algebra in high school results in better preparation for college and improved overall academic success in high school has been disputed by the findings of the studies focused on the implementation of new mandatory curriculum in Chicago Public Schools.

3 Teaching Algebra and Transfer of Knowledge

After examining effects of mandatory algebra in high school, we now turn our attention to impact that this course has on student academic success and graduation rates in higher education. In general, mathematics education should emphasize effective teaching, learning, and transference of mathematics knowledge, regardless of the students' intended major or potential career. In order to accomplish this, the focus of mathematics curricula, especially in algebra, needs to shift from instruction of procedures, to the learning of concepts and solving of complex problems (Lynne, Fendel, Fraser, & Resek, 1997). However, the current methods of teaching algebra, mainly focused on memorization and learning procedures, may favor students who are innately comfortable and skilled with these learning techniques (Gamoran & Hannigan, 2000). The general notion is that we must teach algebra for understanding to ensure lasting knowledge that transfers to new situations (Rittle-Johnson & Alibali, 1999). However, the question of how to teach algebra in order for students to reach that deeper level of comprehension remains unanswered.

Belter (2009) describes a study comparing students' conceptual and procedural knowledge of algebra after receiving either regular instruction or instruction based on a framework for understanding (Belter, 2009). The findings of this study indicated that there were no significant differences in conceptual understanding of algebra between the two groups of students. Overall, regardless of the method of instruction they underwent, students seemed to score lower on conceptual understanding questions than procedural ones, indicating the need "for more effective student-centered implementation of the principles of teaching for understanding" (Belter, 2009, p. 6).

Important questions remain. Namely, *How do we teach algebra to ensure understanding?* Furthermore, *Will identifying appropriate teaching methods aid students in acquiring skills that will make them more likely to succeed in college?* More specifically, *How do we teach algebra to make those learned skills and concepts transferable?* Even if algebra courses were taught in such manner to allow for algebraic skills to transfer or to improve mathematical power of students, there is no evidence in the current literature that supports the belief that any transferable algebra skill contributes to student academic success in non-mathematics courses. The way that algebra is taught is strongly influenced by its purpose - namely, to increase procedural and conceptual knowledge necessary for calculus. If the goal of algebra is now to extend its tools, such as generalization ability and recognizing relationships between quantities, to areas beyond calculus or STEM disciplines, then the objectives of the course, and to a certain degree its content, need to be revisited and possibly reformatted.

There are three foundational understandings that represent obstacles for algebra learning: abstract reasoning, language acquisition, and mathematical (logical) structure (Rakes, Valentine, McGatha, & Ronan, 2010). However, these obstacles to learning transcend the specific mathematics course. Furthermore, some may be present in the majority of college courses, and students who are able to overcome them are more likely to be successful in college. However, there is no evidence that it is this specific course, its objectives, methods of teaching, and learning, that aids students in attaining skills necessary to overcome these challenges.

The benefit and contribution of algebra may be its assistance in helping students transfer their knowledge in novel situations. While there is an extensive research on transfer of learning, the studies that investigate transfer of mathematical knowledge among college students, especially in non-related disciplines, are still very scarce. One of these studies, conducted by Britton (2002), focused on developing an instrument measuring the extent to which students transferred their mathematical skills to new situations in physics, microbiology, and computer science. Student performance on mathematics items was very low, but was higher than on the performance on the transfer task that employed the same mathematical skills. These findings may suggest that students have difficulty applying mathematics in context, and in order for mathematical skills to be transferred, they have to be properly acquired. However, due to the size and type of the sample used in this study (convenience sample of 47 self-selected students), the results and potential implications should not be accepted as widely generalizable.

Another plausible explanation of algebra benefits may lie in its method of instruction, especially if it is conducive to improvement of not only math knowledge, but also transfer of that knowledge. There has been an extensive amount of research done in order to determine what kind of instruction creates a stronger basis for knowledge transfer. For the most part, algebra in higher education and high schools (with the exception of special, inquiry learning based schools) is taught using a traditional, direct instruction method. Although the positive outcomes of these two (direct and inquiry based) most popular styles of teaching and learning have been widely recognized, they could also be supplemented with various other approaches. For example, it was found that self-explanation of material to be learned leads to greater learning and transfer of knowledge. Interestingly, it was the combination of self-explanation and direct instruction, rather than inquiry learning, that led to the greatest procedural learning (Rittle-Johnson, 2006). A technology-based approach to teaching and learning of algebra was also found to have a significantly greater impact on the cognitive performance of students than did a lecture-based approach (Thompson & McCann, 2010).

Although there are studies that direct us toward potential improvements in acquiring knowledge and its transfer, there is no evidence to suggest that this knowledge is transferable beyond mathematics or STEM courses. Even if this were the case, there is no indication that the institutions that require some type of algebra course for all of their students (regardless of their majors) work toward creating mathematics courses that offer the best combination of methods of instruction to ensure the best achievement and transfer outcomes. Again, it is clearly not the subject matter, but the methods of instruction and degree of emphasis on the particular goal of that instructor that is critical.

Anecdotal evidence suggests that students are not able to transfer their mathematics knowledge, especially beyond mathematics courses. Even students who master their mathematics concepts and skills may perform poorly in situations where those skills need to be applied to problems in related disciplines (Britton, 2002). So, if mathematics is not easily transferable even in disciplines incorporating a certain degree of mathematics, what can be expected from students who are required but not prepared to take algebra, and how can they utilize this knowledge in their non-STEM courses? These questions lead us to our final topic: impact of algebra instruction on overall academic achievement as measured by graduation rates.

4 Achievement in Algebra/Math Courses and Graduation Rates in College

It was evident from the extensive analyses of the policy outcomes in Chicago Public Schools that mandatory college preparatory curricula resulted in a decline in high school graduation rates as a direct consequence of students not being able to complete the requirements of the required courses (Mazzeo, 2010). A mandatory algebra course in college may have a similar, negative effect if applied to a broad spectrum of students. A plausible rationale for requiring algebra for all students would be the possibility of acquiring transferable skills that could later assist students in other courses outside of STEM. However, as we have seen from the current literature pertaining to transfer of mathematical knowledge, there is no evidence that algebra aids students in developing such skills.

Requirements to take mathematics courses in college may be based on the philosophical belief that "the study of mathematics offers students another opportunity to discover, create, and communicate knowledge" (Clarion University, 1994). However, a positive relationship between mathematics achievement and student retention through at least four years of college has also been demonstrated empirically (Parker, 2005). The findings indicated that students who graduated in four years were more successful in their required, general education mathematics course. This study does not correct for self-selection bias, but it does note that students who graduated in a timely manner were overall higher-achieving students by stating that "while over 71% of the courses taken by GRAD group [students who graduated in four years] resulted in grades of A or B, only 21% of the WX groups [withdrawn from the university] courses resulted in those grades" (Parker, 2005, p.28). This research supports the belief that students who were more successful in required math courses were more likely to be retained at the University and to graduate in four years. These findings, however, do not support the existence of any causal relationship between success in math courses and student retention and their graduation rates. Simply put, students who succeed in their general education math courses are most likely those students who are overall better prepared and perform higher in not only their math courses, but other general education courses as well.

The benefit of algebra for non-STEM majors, except of acquiring algebra skills, is not clearly supported in the existing literature. If the importance of algebra in college is ensuring well-roundedness of our students in all important aspects of education, and assuming that algebra holds that place over other, non-algebra based mathematics courses or statistics, then this should be clearly stated and grounded in research.

While success in algebra may not be a cause of positive graduation rate, its absence is certainly one of graduation rates prohibitory components. More specifically, it has been reported that the national passing rate in college algebra courses is approximately 40% (Thompson & McCann, 2010). While the existing research, although limited in quantity, directly and indirectly supports the notion that a lack of success in algebra is negatively correlated with graduation rates, there is no empirical evidence that suggests that success in college algebra contributes to the overall college achievement and hence improvement in graduation rates. The problem is not that we just need to find some way for all students to succeed at an algebra course and then they will succeed in college. Rather, the issue is that the lack of student preparation most easily is seen in their inability to succeed in algebra.

5 Conclusions

College algebra may be an indicator of student success in college, but there is no evidence that this subject contributes to that success. Students who succeed in algebra are likely to already have the ability and skill to meet the requirements of not only this course, but also the requirements of their other general education and major specific courses, and hence graduate in a timely manner. There is no evidence in the literature that suggests that the context or structure of algebra, or any mathematics course for that matter, aids students in developing or even improving any of their cognitive skills, but only their content knowledge. While it is not impossible that this could be the case in a particular situation if the mathematics curricula were constructed with this in mind, no such studies were found in the literature.

The "algebra for everyone" attitude may be justified in high school, but this may also be true of many other courses, such as sciences, English, and humanities. These are all integral courses that we want our students to be successful in as we try to adequately prepare them for higher education and their future careers. Having this purpose may even explain why algebra would be required or at least strongly recommended in college. This purpose, however, should not be confused with the common perception that success in algebra in some way increases a student's general achievement and success in college. The relationship between success in algebra and success in college is not necessarily causal. Is it possible that if reformed with this purpose in mind algebra could be taught in a manner that could impact key components of overall success? The literature suggests that this might be possible to some small (as yet unquantifiable) extent, but only if we are willing to engage in targeted research to inform such a major reform. Therefore, either we continue with the existing format of algebra, in terms of its benefits, curriculum, structure, and method of instruction, or we modify it so that all students benefit from this subject. Only in this way can we make this course into one that is partially justified with the words: "you can live without it, but you will not appreciate as much of what is going on around you" (Usiskin, 1995, p.37).

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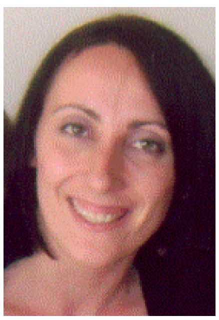

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